SECTION ONE - (3 point problems)

1. 5^8 pencils will be shared to 25 empty boxes equally. How many pencils will be in each box?

   (A) 25^2 (B) 25^3 (C) 5^3

   (D) 5^2 (E) 5

2. How many positive three-digit integers are divisible by 13?

   (A) 68   (B) 69   (C) 70
   (D) 76   (E) 77

3. Bella is older than Charlie and younger than Lily. Teddy is older than Bella. Which two people could be the same age?

   (A) Charlie and Teddy   (B) Teddy and Lily
   (C) Lily and Charlie   (D) Bella and Lily
   (E) Teddy and Bella

4. The product of the digits of a 10-digit integer is 15. What is the sum of the digits of this number?

   (A) 8   (B) 12   (C) 15
   (D) 16   (E) 20

5. Four circles, each of radius 1, intersect as shown. What is the perimeter of the shaded region?

   (A) π

   (B) Some number between $\dfrac{3\pi}{2}$ and 2π

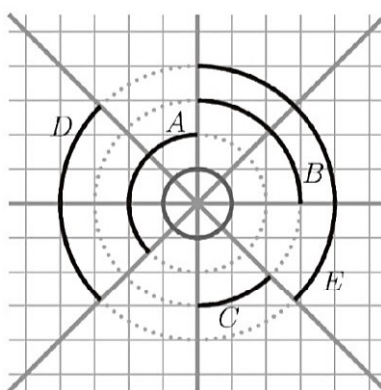   (C) $\dfrac{3\pi}{2}$

   (D) 2π

   (E) π^2

6. David writes, in increasing order, all the integers from 2 to 2022 which use only 0s and 2s. What is the number in the middle of his list?

   (A) 200   (B) 220   (C) 222
   (D) 2000   (E) 2002

7. How many real solutions does the equation $(x - 2)^2 + (x + 2)^2 = 0$ have?

   (A) 0   (B) 1   (C) 2
   (D) 3   (E) 4

8. Four lines intersect forming eight equal angles. Which black arc has the same length as the small grey circle?

   (A) A   (B) B   (C) C
   (D) D   (E) E

9. Let a, b, c be non-zero numbers. The numbers $-2a^4b^3c^2$ and $3a^3b^5c^{-4}$ have the same sign. Which of the following is definitely true?

   (A) $ab > 0$ (B) $b < 0$ (C) $c > 0$
   (D) $bc > 0$ (E) $a < 0$

10. Mike has marked the points A, B, C and D in this order on a straight line, as shown in the diagram.

   The distance between A and C is $12\ cm$ and between B and D, $18\ cm$. What is the distance between the midpoint of AB and the midpoint of CD?

   (A) $15\ cm$ (B) $12\ cm$ (C) $18\ cm$
   (D) $6\ cm$ (E) $9\ cm$

SECTION TWO - (4 point problems)

11. When he looks at the water meter in his bathroom, Tony notices that all the digits on the meter are different.

$$\boxed{9}\,\boxed{1}.\boxed{8}\,\boxed{7}\,\boxed{6}\ \mathrm{m}^3$$
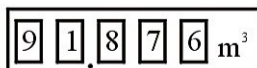
How much water will be used until the next time all the digits on the meter are different?

(A) $0.006m^3$ (B) $0.034m^3$ (C) $0.086m^3$
(D) $0.137m^3$ (E) $1.048m^3$

12. A large square is divided into two unequal squares and two equal rectangles, as shown. The vertices of the shaded quadrilateral are the midpoints of the sides of the two squares. The area of the shaded quadrilateral is 3. What is the area of the unshaded part of the large square?
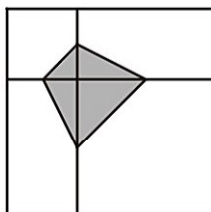
(A) 12   (B) 15   (C) 18
(D) 21   (E) 24

13. What is the greatest common divisor of $2^{2021} + 2^{2022}$ and $3^{2021} + 3^{2022}$?

(A) 2^{2021} (B) 1   (C) 2
(D) 6   (E) 12

14. The map shows a region with 16 cities connected by roads. The Government wants to build electricity power plants in some of the cities. Each power plant can provide enough electricity for the city where it is sited and any cities connected to that city by a single road. What is the smallest number of power plants that need to be built?
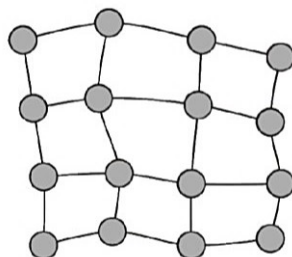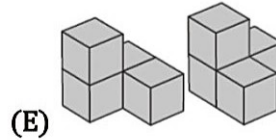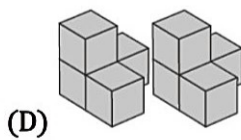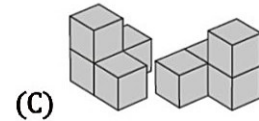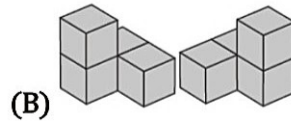

(A) 3   (B) 4   (C) 5
(D) 6   (E) 7

15. Which of the pairs of pieces below can be put together to build the shape shown in the diagram?

(A)

(B)

(C)

(D)

(E)
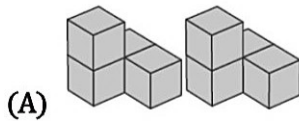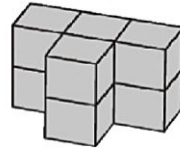
16. Martina is playing in an 8 player tournament. She knows she will beat everyone except Ash, who will beat everybody. In the first round, players are organized randomly into four pairs, and the winner of each match proceeds to the second round. In the second round, there are two matches and the winners of these matches proceed to the final. What is the probability that Martina does not get to the final?

(A) 1 (B) 1/2 (C) 2/7
(D) 3/7 (E) 4/7

17. A cuboid of surface area S is cut by six planes as shown. Each plane is parallel to a face, but its distance from the face is random. Now the cuboid is separated in **27** smaller parts. What, in terms of S, is the total surface area of all **27** smaller parts?
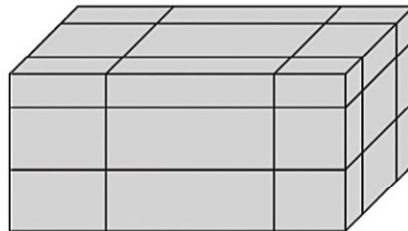
(A) $2\,S$ (B) $\frac{5}{2}S$ (C) $3\,S$
(D) $4\,S$ (E) none of the previous

18. Five numbers have a mean of 24. The mean of the three smallest numbers is 19 and the mean of the three largest numbers is 28. What is the median of the five numbers?
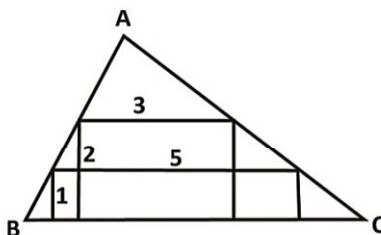
(A) 20 (B) 21 (C) 22
(D) 23 (E) 24

19. Two rectangles are inscribed inside a triangle ABC. The dimensions of the rectangles are 1×5 and 2×3, as shown. What is the height of the triangle with base BC?


(A) 3 (B) $\frac{7}{2}$ (C) $\frac{8}{3}$

(D) $\frac{16}{5}$ (E) none of the previous

20. A rectangle is divided into 11 smaller rectangles, as shown in the diagram. All 11 rectangles are similar to the original large rectangle. The orientation of the smallest rectangles is the same as the largest. The length of the base of the smallest rectangle is 1. What is the perimeter of the original rectangle?
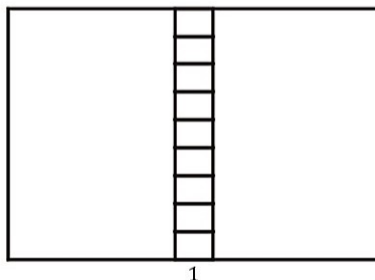
(A) 20 (B) 24 (C) 27
(D) 30 (E) 36

SECTION THREE - (5 point problems)

21. A circle with centre (0, 0) has radius 5. At how many points on the perimeter of the circle are both coordinates integers?
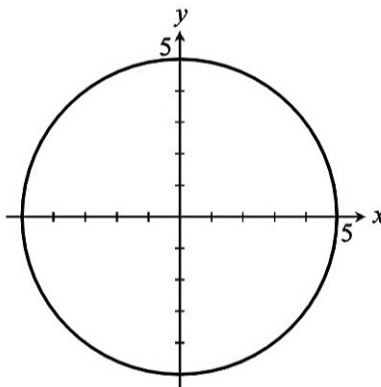
(A) 5 (B) 8 (C) 12
(D) 16 (E) 20

22. How many positive three-digit integers are there that are equal to five times the product of their digits?

(A) 1   (B) 2   (C) 3
(D) 4   (E) 5

23. The numbers 1 to 10 are placed, once each, in the circles of the figure shown. The sum of the numbers in the left column is 24; the sum of the numbers in the right column is also 24 and the sum of the numbers in the bottom row is 25. What number is in the circle containing the question mark?
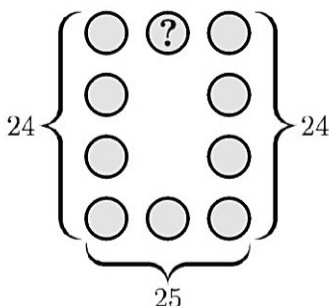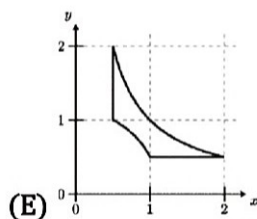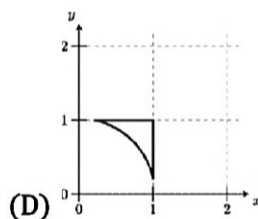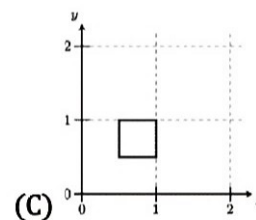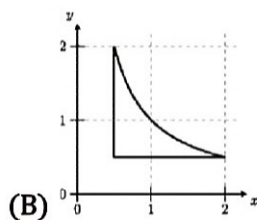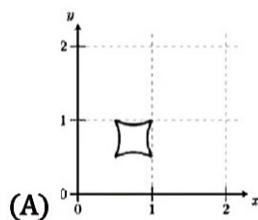

(A) 2   (B) 4   (C) 5
(D) 6   (E) none of the previous
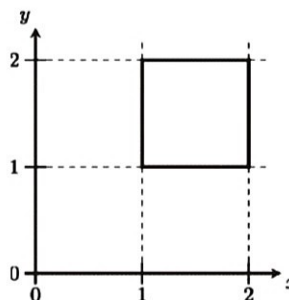
24. A square lies in a coordinate system as shown.

Each point (x, y) on the square is moved to $\left(\dfrac{1}{x}, \dfrac{1}{y}\right)$. What will the resulting figure look like?

(A)

(B)

(C)

(D)

(E)

25. The vertices of a 20-gon are numbered from 1 to 20 in such a way that the numbers of adjacent vertices differ by either 1 or 2. The sides of the 20-gon whose ends differ by only 1 are colored red. How many red sides are there?

(A) 1   (B) 2   (C) 5
(D) 10   (E) there are multiple possibilities

26. Two circles cut a rectangle $AFMG$, as shown. The line segments outside the circles have length $AB = 8, CD = 26, EF = 22, GH = 12$ and $JK = 24$. What is the length of LM?
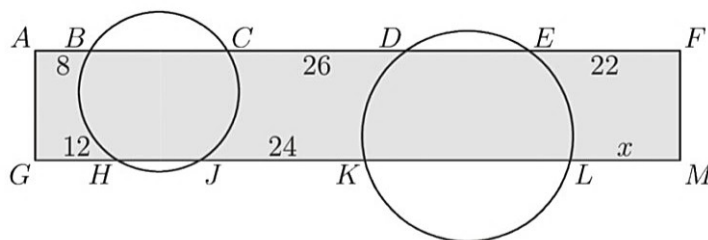
(A) 14   (B) 15   (C) 16
(D) 17   (E) 18

27. Let N be a positive integer. How many integers are there between
$$\sqrt{N^2 + N + 1} \text{ and } \sqrt{9N^2 + N + 1}?$$

(A) N + 1   (B) 2N − 1   (C) 2N
(D) 2N + 1   (E) 3N

28. In a sequence, the first term, a_1 is between 0 and 1. For all $n \geq 1, a_{2n} = a_2 \cdot a_n + 1$ and $a_{2n+1} = a_2 \cdot a_n - 2$. Given that $a_7 = 2$, what is the value of a_2?

(A) Equal to a_1 (B) 2   (C) 3
(D) 4   (E) 5

29. A regular hexagonal prism has its top corners shaved off, as shown. The top face becomes a smaller regular hexagon and the 6 rectangular faces around the middle become 12 isosceles triangles of two different sizes. What fraction of the volume of the original prism has been lost?
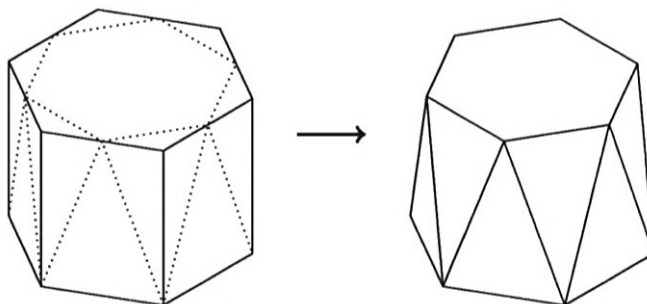
(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4\sqrt{3}}$

(D) $\frac{1}{6\sqrt{2}}$ (E) $\frac{1}{6\sqrt{3}}$

30. A football match between teams from North Berracan and South Berracan is played in a stadium that has a rectangular array of seats for the spectators. There are 11 North Berracan supporters in each row, and 14 South Berracan supporters in each column. This leaves 17 empty seats. What is the smallest possible number of seats in the stadium?

(A) 500   (B) 660   (C) 690
(D) 840   (E) 994