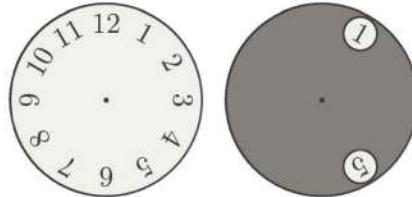# 33ʳᵈ INTERNATIONAL KANGAROO MATHEMATICS CONTEST 2023

KSF - Problems Junior (Class 9 & 10)

Time Allowed: 180 minutes

SECTION ONE - (3 point problems)

1. A grey circle with two holes is put on top of a clock-face, as shown.
   The grey circle is turned around the centre such that the number 10 appears in one hole. Which numbers is it possible to see in the other hole?

(A) 2 and 6　　　　(B) 3 and 7　　　　(C) 3 and 6
(D) 1 and 9　　　　(E) 2 and 7

2. Maria had to run to catch the subway, got off two stops later and then walked to school. Which of the following speed-time graphs would best represent her journey?

(A)　　　　(B)　　　　(C)

(D)　　　　(E)
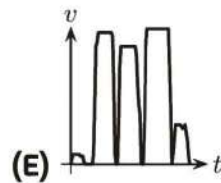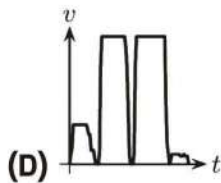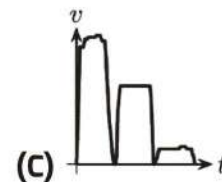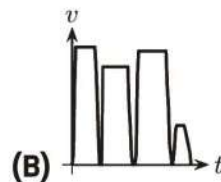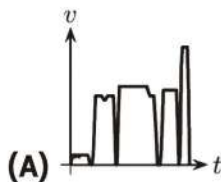
3. The positive integers m and n are both odd. Which of the following integers is also odd?

(A) $m(n+1)$　　　　(B) $(m+1)\cdot(n+1)$　　　　(C) $m+n+2$
(D) $m\cdot n+2$　　　　(E) $m+n$

4. A large square of side-length $10\ cm$ contains a smaller square of side-length $4\ cm$, as shown in the diagram. The corresponding sides of the two squares are parallel. What percentage of the large square is shaded?

(A) 25%　　　　(B) 30%
(C) 40%　　　　(D) 42%
(E) 45%

5. **Today is Thursday. What day will it be in 2023 days' time?**

    (A) Tuesday   (B) Wednesday   (C) Thursday
    (D) Friday   (E) Saturday

6. **The large rectangle in the diagram is divided into 30 equal squares, as shown.**
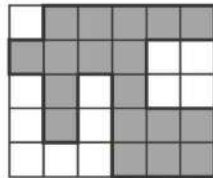
    The perimeter of the shaded region is 240 cm. What is the area of the rectangle?

    (A) 480 cm^2 (B) 750 cm^2 (C) 1080 cm^2
    (D) 1920 cm^2 (E) 2430 cm^2

7. **The ages of a family of five add to 80. The two youngest are 6 and 8. What was the sum of the ages of the family seven years ago?**

    (A) 35   (B) 36   (C) 45
    (D) 46   (E) 66

8. **A wooden fence consists of a series of vertical planks, each joined to the next post by four horizontal planks. The first and last plank in the fence are vertical. Which of the following could be the total number of planks in the fence?**

    (A) 95   (B) 96   (C) 97
    (D) 98   (E) 99

9. **The letters a and b are to be replaced by positive integers so that the equation is correct.**

$$\frac{a}{5} = \frac{7}{b}$$

    In how many different ways can this be done?

    (A) 0   (B) 1   (C) 2
    (D) 3   (E) 4

10. **After having played 200 games of chess, my winning rate is exactly 49%. What is the smallest number of extra games I need to play to increase my winning rate to exactly 50%?**

    (A) 0   (B) 1   (C) 2
    (D) 3   (E) 4

SECTION TWO - (4 point problems)

11. Jenni is trying to save water. She reduced the time she spent in her shower by a quarter. She also lowered the water pressure of her shower to reduce the rate the water comes out of the shower head by a quarter. By what fraction did Jenni reduce the total amount of water she uses for a shower?

(A) $by \frac{1}{4}$ (B) $by \frac{3}{8}$ (C) $by \frac{5}{8}$

(D) $by \frac{5}{12}$ (E) $by \frac{7}{16}$

12. The diagram shows three squares of side-length $3\,cm$, $5\,cm$ and $8\,cm$. What is the area, in cm^2, of the shaded trapezium?
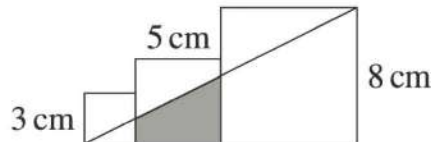
(A) 13 (B) $\frac{55}{4}$ (C) $\frac{61}{4}$

(D) $\frac{65}{4}$ (E) $\frac{69}{4}$

13. A wire of length $95\,m$ is cut into three pieces such that the length of each piece is 50% more than the previous piece. What is the length of the largest piece?

(A) $36\,m$ (B) $42\,m$ (C) $45\,m$
(D) $46\,m$ (E) $48\,m$

14. Points M and N are the midpoints of two sides of the rectangle. What fraction of the area of the rectangle is shaded?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$

(D) $\frac{1}{3}$ (E) $\frac{1}{2}$

15. Pentagon $ABCDE$ is divided into four triangles with equal perimeter. Triangle ABC is equilateral and AEF, DFE and CDF are three identical isosceles triangles. What is the ratio of the perimeter of the pentagon $ABCDE$ to the perimeter of triangle ABC?

(A) 2/1   (B) 3/2   (C) 4/3
(D) 5/3   (E) 5/2

16. On the table there is a tower made of blocks numbered from 1 to 90. Bob takes blocks from the top of the tower, three at a time, to build a new tower, as shown. When he has finished building the new tower, how many blocks will be between the blocks numbered 39 and 40?

(A) 0   (B) 1
(C) 2   (D) 3
(E) 4

17. Every third step of a staircase with 2023 steps is coloured black. The first seven steps are shown in the diagram. Anita walks up the steps one at a time, starting with either her right or left foot, alternating each step. What is the smallest number of black steps she will step on with her right foot?

(A) 0   (B) 333   (C) 336
(D) 337   (E) 674

18. We call a two-digit number power-less if none of its digits can be written as an integer to a power greater than 1. For example, 53 is power-less, but 54 is **NOT** power-less since $4 = 2^2$. Which of the following is a common divisor of the smallest and the largest power-less numbers?

(A) 3   (B) 5   (C) 7
(D) 11   (E) 13

19. A square of side $30\,cm$ is divided into nine identical smaller squares. The large square contains three circles with radii $5\,cm$ (bottom right), $4\,cm$ (top left) and $3\,cm$ (top right), as shown. What is the area of the shaded part?
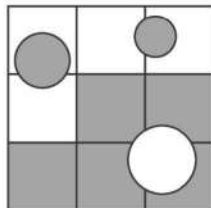
(A) $400\,cm^2$ (B) $500\,cm^2$ (C) $(400 + 50\pi)cm^2$
(D) $(500 - 25\pi)cm^2$ (E) $(500 + 25\pi)cm^2$

20. Tim calculates the mean of five different prime numbers. His answer is an integer. What is the smallest possible integer he could have obtained?

(A) 2 (B) 5 (C) 6
(D) 12 (E) 30

SECTION THREE - (5 point problems)

21. The diagram shows two touching semicircles of radius 1 and parallel diameters AB and CD. What is the square of the distance AD?
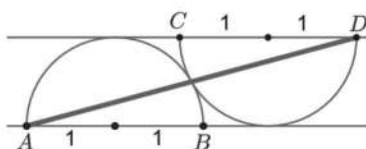
(A) 16 (B) $8 + 4\sqrt{3}$ (C) 12
(D) 9 (E) $5 + 2\sqrt{3}$

22. When it is given a list of four numbers, the Kangaroo Machine continues the list by typing the smallest non-negative integer that is different to each of the four preceding terms and then repeats this process over and over again. Jacob types in the numbers

$$2, 0, 2, 3,$$

into the machine. What number will be the 2023ʳᵈ in the list?

(A) 0 (B) 1 (C) 2
(D) 3 (E) 4

23. A rectangle with vertices (0, 0), (100, 0), (100, 50) and (0, 50) has a circle with centre (75, 30) and radius 10 cut out of it. What is the gradient of the line through (75, 30) that divides the remaining area of the rectangle into two equal parts?

(A) $\frac{1}{5}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{5}$

(E) $\frac{2}{3}$

24. When Metin's phone is fully charged, it runs out in 32 hours if he only uses it for phone-calls, in 20 hours if he only uses it for the internet, and in 80 hours if he does not use it at all. Metin gets on a train with his phone half-charged. While on the train, the time he is on the internet, the time he is making phone-calls and the time he is not using it are all the same. His phone runs out of charge just as the train reaches his destination. How many hours did the train journey take?

(A) 10

(B) 12

(C) 15

(D) 16

(E) 18

25. Seven different single-digit numbers are written in the circles of the diagram shown with one number in each circle. The product of the three numbers in each of the three lines of three numbers is the same. Which number is written in the circle containing the question mark?
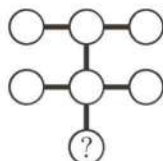
(A) 2

(B) 3

(C) 4

(D) 6

(E) 8

26. Leon has drawn a closed path on a cuboid and then unfolded it to give a net. Which of the nets shown could not be the net of Leon's cuboid?

(A)

(B)

(C)

(D)

(E)

27. How many three-digit positive integers x are there, such that subtracting the sum of digits of x from x gives a three-digit number whose digits are all the same?

(A) 1   (B) 2   (C) 3
(D) 20   (E) 30

28. In how many different ways can the word **BANANA** be read from the following table by moving from one cell to another cell with which it shares an edge? Cells may be visited more than once.

B	A	N
A	N	A
N	A	N

(A) 14   (B) 28   (C) 56
(D) 84   (E) other value

29. The diagram shows a map of a park. The park is divided into regions. The number inside each region gives its perimeter, in km. What is the outer perimeter of the park?

(A) 22 km (B) 26 km
(C) 28 km (D) 32 km (E) none of the previous

30. Pia wants to write the integers 1 to 9 in the nine boxes shown so that the integers in any three adjacent boxes add to a multiple of 3. In how many ways can she do this?

(A) 6^4 (B) 6^3 (C) 2^9
(D) 6×5×4×3×2×1   (E) 9×8×7×6×5×4×3×2×1

-- Good Luck --