

33rd INTERNATIONAL KANGAROO MATHEMATICS CONTEST 2023

KSF - Problems Student (Class 11, 12 & 13)

Time Allowed: 180 minutes

SECTION ONE - (3 point problems)

1. What is the value of $\frac{7777^2}{5555 \times 2222}$?

- (A) 1 (B) $\frac{7}{10}$ (C) $\frac{49}{10}$
(D) $\frac{77}{110}$ (E) 49

2. Giulia rolls five dice. She rolls 19 points in total. What is the maximum number of sixes she could have rolled?

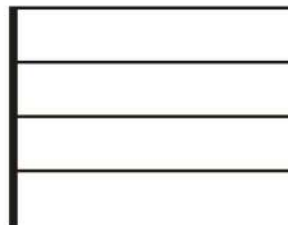
- (A) 0 (B) 1 (C) 2
(D) 3 (E) 4

3. A cylindrical can has height 15 cm and the perimeter of its circular base is 30 cm. An ant walks from point A on the base to point B on the roof. Its path is either vertically upwards or horizontally along circular arcs around the can. Its path is shown with a thicker line (black for the path on the front of the can and grey at the back). What is the length, in cm, of the ant's path?



- (A) 45 (B) 55 (C) 60
(D) 65 (E) 75

4. Emma has four different coloured pens. She wants to colour the three-striped rectangular flag shown in the diagram so that each stripe is a single colour and no two adjacent stripes are the same colour. In how many ways can she do this?



- (A) 24 (B) 27 (C) 32
(D) 36 (E) 64

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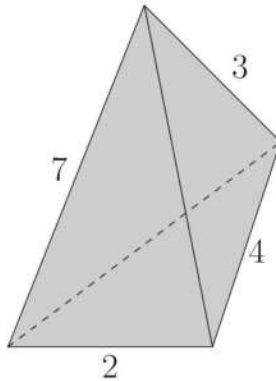
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10. What is the units digit of the product $(5^5 + 1)(5^{10} + 1)(5^{15} + 1)$?

- (A) 1 (B) 2 (C) 4
(D) 5 (E) 6

SECTION TWO - (4 point problems)

11. A triangular pyramid has edges of integer length. Four of these lengths are as shown in the diagram. What is the sum of the lengths of the other two edges?



- (A) 9 (B) 10 (C) 11
(D) 12 (E) 13

12. For a positive integer n , $n!$ is defined as the product of all integers from 1 to n . For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. What is the sum of the digits of N if $N! = 6! \cdot 7!$?

- (A) 1 (B) 2 (C) 4
(D) 8 (E) 9

13. The graphs of the functions $y = x^3 + 3x^2 + ax + 2a + 4$ all pass through the same point, no matter what value of a is chosen. What is the sum of the coordinates of that point?

- (A) 2 (B) 4 (C) 7
(D) 8 (E) none of the previous

14. We are given five numbers a_1, a_2, a_3, a_4, a_5 whose sum is S . For each $k, 1 \leq k \leq 5$, we know that $a_k = k + S$. What is the value of S ?

- (A) $\frac{15}{4}$ (B) $-\frac{15}{4}$ (C) -15
(D) 15 (E) none of the previous

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15. How many pairs of integers m and n satisfy the inequality

$$|2m - 2023| + |2n - m| \leq 1?$$

- (A) 0 (B) 1 (C) 2
(D) 3 (E) 4

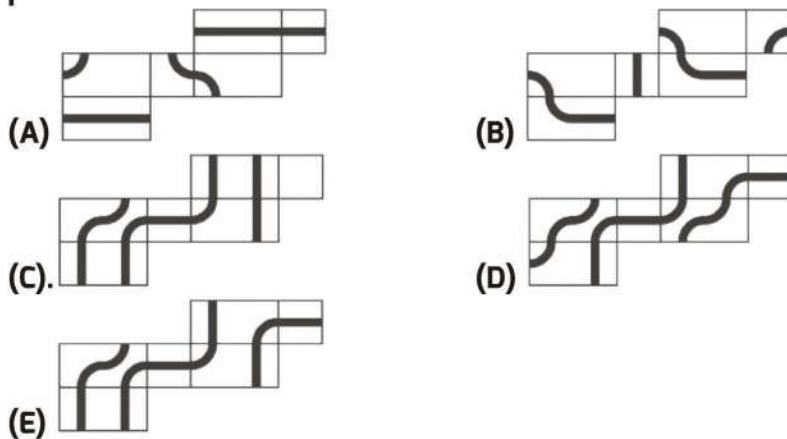
16. There are 23 animals sitting in a row at the cinema. Each animal is either a beaver or a kangaroo. Everyone has at least one neighbour who is a kangaroo. What is the largest possible number of beavers in the row?

- (A) 7 (B) 8 (C) 10
(D) 11 (E) 12

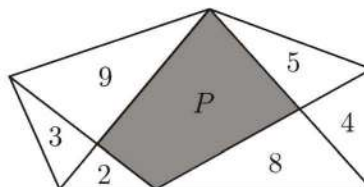
17. The number 5^{56} can be written in the form n^n for some integer n . What is the value of n ?

- (A) 5^{30} (B) 5^6 (C) 5^5
(D) 30 (E) 11

18. Leon has drawn a closed path on a rectangular prism. Which net could show his path?

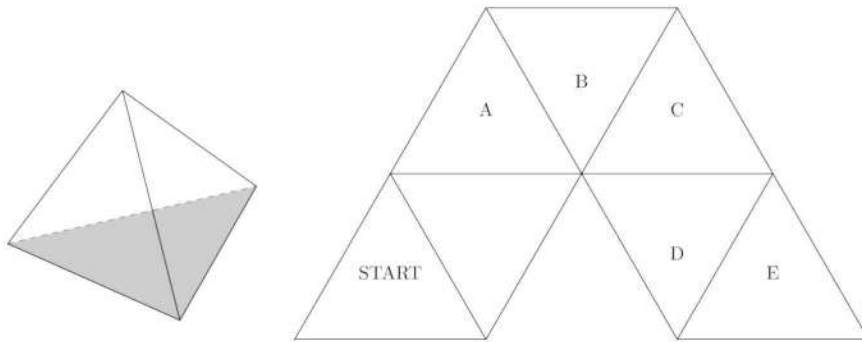


19. A pentagon is dissected into smaller parts, as shown. The numbers inside the triangles indicate their areas. What is the area P of the shaded quadrilateral?



- (A) 15 (B) $\frac{31}{2}$ (C) 16
(D) 17 (E) 18

24. A block in the shape of a regular tetrahedron has one face shaded.

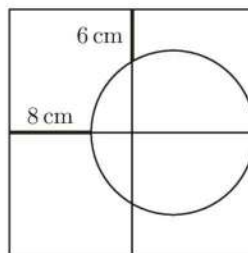


The shaded face of the block is placed on the board on the triangle labelled START. The block is then rolled from one triangle to the next by rotating it about one edge. On which triangle will the block stand for the first time again on its shaded face?

- (A) A (B) B (C) C
 (D) D (E) E
25. Part of the fifth degree polynomial shown cannot be seen because of an inkblot. It is known that all five roots of the polynomial are integers. What is the highest power of $x - 1$ that divides the polynomial?

$$x^5 - 11x^4 + \text{[inkblot]} - 7$$

- (A) $(x - 1)^1$ (B) $(x - 1)^2$ (C) $(x - 1)^3$
 (D) $(x - 1)^4$ (E) $(x - 1)^5$
26. The large square in the diagram is dissected into four smaller squares. The circle touches the right hand side of the square at its midpoint. What is the side-length of the large square? Note that the diagram is not drawn to scale.



- (A) 18 cm (B) 20 cm (C) 24 cm
 (D) 28 cm (E) 30 cm

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-- Good Luck --