Student Level: Class (11, 12 & 13) **Max Time: 2 Hours**

3-Point Problems

1. There are 200 fish in an aquarium. 1 % of them is blue, all the rest are yellow. How many yellow fish do we have to take out the aquarium, so that blue fish represent 2 % of all aquarium fish?

(**A**) 2 (**B**) 4 (**C**) 20 (**D**) 50 (**E**) 100

2. Which is the largest of the following numbers?

(**A**) $\sqrt{2}-\sqrt{1}$ (**B**) $\sqrt{3}-\sqrt{2}$ (**C**) $\sqrt{4}-\sqrt{3}$ (**D**) $\sqrt{5}-\sqrt{4}$ (**E**) $\sqrt{6}-\sqrt{5}$

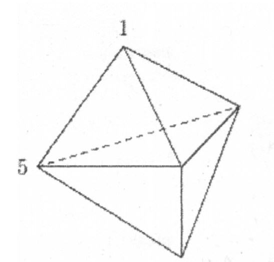
3. For how many different positive integers n is the number n^2+n a prime number?

(**A**) 0 (**B**) 1
(**C**) 2 (**D**) a finite number but more than 2
(**E**) an infinite number

4. Mari, Ville and Ossi went to a café. Each of them bought three glasses of juice, two icecreams and five buns. Which of the following could be the total bill?

(**A**) 39.20 (**B**) 38.20 (**C**) 37.20 (**D**) 36.20 (**E**) 35.20

5. The picture shows a solid formed with 6 triangular faces. At each vertex there is a number. For each face we consider the sum of the 3 numbers at the vertices of that face. If all the sums are the same and two of the numbers are 1 and 5 as shown, what is the sum of all the 5 numbers?



(**A**) 9 (**B**) 12 (**C**) 17 (**D**) 18 (**E**) 24

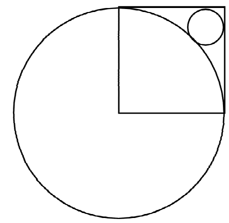
6. Circles $f(F;13)$, $g(G;15)$ intersect in points P, Q. Length of line segment PQ is 24. Which of the following could be the length of the segment FG?

(**A**) 2 (**B**) 5 (**C**) 9 (**D**) 14 (**E**) 18

7. A box contains 2 white, 3 red and 4 blue socks. Liz knows, that a third of the socks have a hole in them but not what colour the worn through socks are. She is picking up socks from the box to the floor at random and hoping to get two good socks of the same colour. How many socks must she take to be absolutely sure to have a good pair?

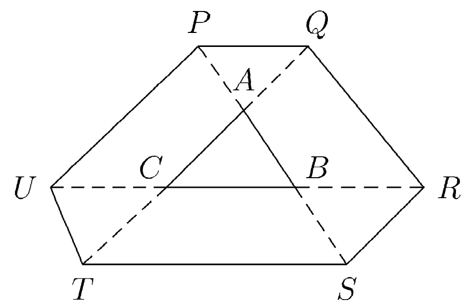
(**A**) 2 (**B**) 3 (**C**) 6 (**D**) 7 (**E**) 8

8. The square in the figure has side equal to 1. Then the radius of the small circle is equal to



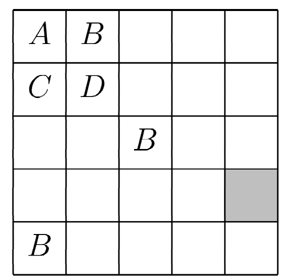
(**A**) $\sqrt{2}-1$ (**B**) $\frac{1}{4}$ (**C**) $\frac{\sqrt{2}}{4}$ (**D**) $1-\frac{\sqrt{2}}{2}$ (**E**) $(1-\sqrt{2})^2$

9. Sides of triangle ABC are continued to both sides to points P, Q, R, S, T and U so that $|PA| = |AB| = |BS|$, $|TC| = |CA| = |AQ|$ and $|UC| = |CB| = |BR|$. If the area of ABC is 1, what is the area of the hexagon $PQRSTU$?



(**A**) 9 (**B**) 10 (**C**) 12
(**D**) 13 (**E**) not enough information

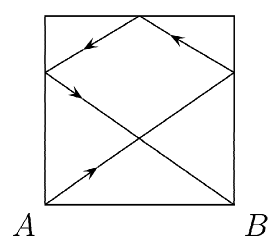
10. We want to colour the squares in the grid using colors A, B, C and D in such a way that neighboring squares do not have the same colour (squares that share a vertex are considered neighbors). Some of the squares have been coloured as shown. What are the possibilities for shaded square?

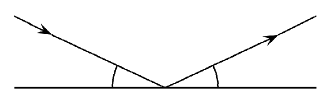


(**A**) any of A or B (**B**) only C (**C**) only D
(**D**) any of C or D (**E**) any of A, B, C, D

4-Point Problems

11. On a square shaped billiard table with side 2 m, a ball is thrown from the corner A. After touching three sides as shown it goes to corner B. How many meters did the ball travel? (Remember that a ball bounces with the same angle that it enters as shown in the picture on the right.)



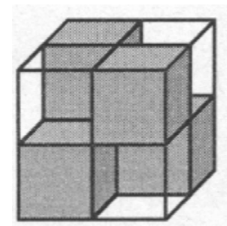


(**A**) 7 (**B**) $2\sqrt{13}$ (**C**) 8 (**D**) $4\sqrt{3}$ (**E**) $2(\sqrt{2}+\sqrt{3})$

12. 2009 kangaroos, each of them either light or dark, compare their heights. It is known that one light kangaroo is higher than exactly 8 dark kangaroos, one light kangaroo is higher than exactly 9 dark kangaroos, one light kangaroo is higher than exactly 10 dark kangaroos, and so on, and exactly one light kangaroo is higher than all dark kangaroos. What is the number of light kangaroos?

(**A**) 1000 (**B**) 1001 (**C**) 1002
(**D**) 1003 (**E**) this situation is impossible

13. A cube measuring $2 \times 2 \times 2$ is formed of four $1 \times 1 \times 1$ white transparent and four $1 \times 1 \times 1$ black non-transparent cubes (picture). They are placed in the way that the whole big cube is non-transparent, meaning that it is not possible to see through it neither from top to bottom, nor from front to back and not even from left to right. At least how many black cubes would we have to put into the big cube measuring $3 \times 3 \times 3$ to make the whole cube non-transparent?



(A) 6 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 18

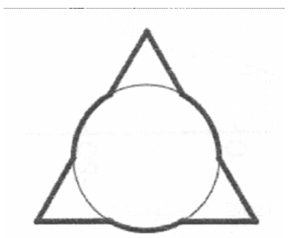
14. On the island of nobles and liars 25 people are standing in a queue. Everyone, except the first person in the queue, said, that the person before him in the queue is a liar, and the first man in the queue said, that all people, standing after him are liars. How many liars are there in the queue? (Nobles always speak the truth, and liars always tell lies.)

(A) 0 **(B)** 12 **(C)** 13
(D) 24 **(E)** impossible to determine

15. What is the last digit of the number $1^2 - 2^2 + \ldots - 2008^2 + 2009^2$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

16. We overlap an equilateral triangle with side length of 3 and a circle of radius 1 matching the centers of the two figures. What is the length of the perimeter of the figure that we get?



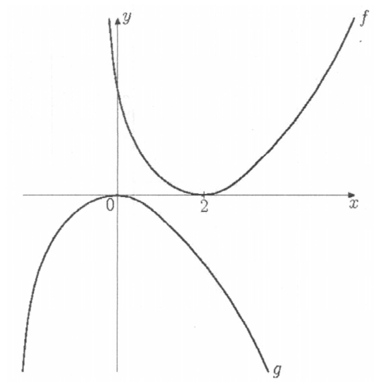
(A) $3 + 2\pi$ **(B)** $6 + \pi$ **(C)** $9 + \dfrac{\pi}{3}$ **(D)** 3π **(E)** $9 + \pi$

18. Four problems were proposed to each of 100 contestants of a Mathematical Olympiad. 90 contestants solved the first problem, 85 contestants solved the second problem, 80 contestants solved the third problem, and 70 contestants solved the fourth problem. What is the smallest possible number of the contestants which solved all four problems?

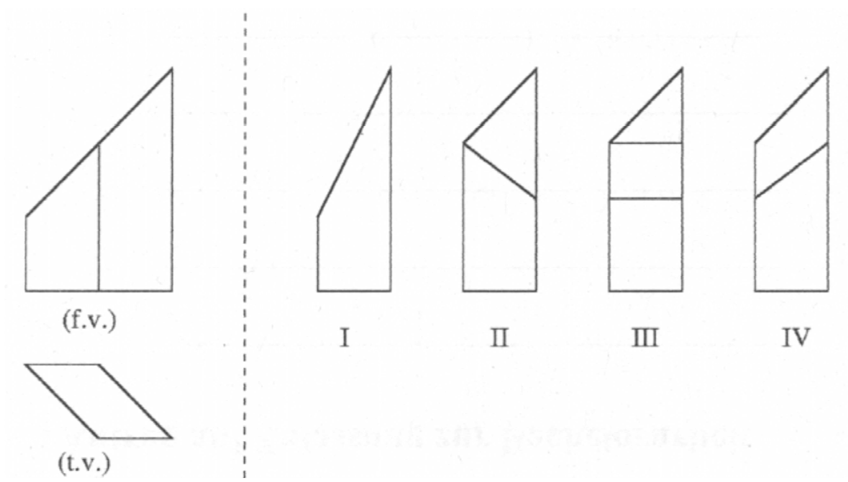
(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

17. Graphs of real functions f and g are on the figure. What is the relation between f and g?

(**A**) $g(x) = f(x+2)$
(**B**) $g(x-2) = -f(x)$
(**C**) $g(x) = -f(-x+2)$
(**D**) $g(-x) = -f(-x+2)$
(**E**) $g(2-x) = -f(x)$



19. In the figure below you see the front view (f.v.) and the top view (t.v.) of a geometric solid.



Which of the figures I to IV describes the view from the left?

(**A**) Figure I (**B**) Figure II (**C**) Figure III (**D**) Figure IV (**E**) None of the above

20. We have constructed a 3×3-squaretable of real numbers in which the sum in each row, column and diagonal is the same. Two of the numbers are shown in the figure. Which number must be in position a?

a		
		47
	63	

(**A**) 16 (**B**) 51 (**C**) 54
(**D**) 55 (**E**) 110

5-Point Problems

21. Two runners A and B are going round a stadium. Each of them are running all the time at the same speed. A runs faster than B and it takes 3 minutes to A for one turn. A and B start together and 8 minutes further, A catches B up for the first time. How long does it take to B for one turn?

(**A**) 6 min (**B**) 8 min (**C**) 4 min 30 sec
(**D**) 4 min 48 sec (**E**) 4 min 20 sec

22. Let Z be the number of 8-digit numbers with 8 different digits, none of which is 0. How many 8-digit numbers exist that are divisible by 9, that have 8 different digits, none of which is 0?

(**A**) $\frac{Z}{8}$ (**B**) $\frac{Z}{3}$ (**C**) $\frac{Z}{9}$
(**D**) $\frac{8Z}{9}$ (**E**) $\frac{7Z}{8}$

23. How many ten-digit numbers only composed of 1, 2 and 3 exist, in which any two neighboring digits differ by 1.

(**A**) 16 (**B**) 32 (**C**) 64 (**D**) 80 (**E**) 100

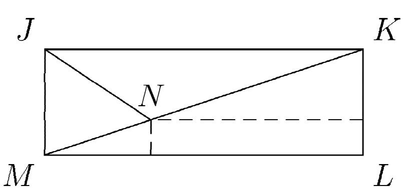
24. For how many integers $n \geq 3$ does there exists a convex n-gon, whose angles are in ratio $1 : 2 : \ldots : n$?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 5 (**E**) more than 5

25. 55 schoolchildren took part in math Olympiad. When checking the problems, the jury marked them either with "+" the problem was solved, or with "–" the problem was not solved, or with "0" participant skipped the problem. Later it occurred that no two works had the same number of "+" and "–". What is the least number of problems at the Olympiad?

(**A**) 6 (**B**) 9 (**C**) 10
(**D**) 11 (**E**) 12

26. In a rectangle $JKLM$, the bisector of angle KJM cuts the diagonal KM at point N. The distances between N and sides LM and KL are respectively 1 and 8. Then LM is:



(**A**) $8+2\sqrt{2}$ (**B**) $11-\sqrt{2}$ (**C**) 10 (**D**) $8+3\sqrt{2}$ (**E**) $11+\frac{\sqrt{2}}{2}$

27. If $k = \dfrac{a}{b+c} = \dfrac{b}{c+a} = \dfrac{c}{a+b}$, how many possible values of k are there?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 6

28. The numbers $1; 2; 3; \ldots; 99$ are distributed into n groups under the conditions:

1. each number is exactly in one group;
2. there are at least two numbers in each group;
3. if two numbers are in one and the same group, then their sum is not divisible by 3.

The smallest n with this property is:

(**A**) 3 (**B**) 9 (**C**) 33 (**D**) 34 (**E**) 66

29. Samantha and her 3 sisters go to the theatre. They have a booth with four seats. Samantha and two of her sisters arrive earlier and each take a seat at random on any of the four seats. What is the probability that Samantha has to move when her youngest sister Marie arrives if Marie insists that she take her assigned seat and then so does any of the sisters that have to stand up.

(**A**) $\dfrac{3}{4}$ (**B**) $\dfrac{1}{2}$ (**C**) $\dfrac{1}{3}$ (**D**) $\dfrac{1}{4}$ (**E**) $\dfrac{1}{6}$

30. The sequence of integers a_n is defined by: $a_0 = 1$, $a_1 = 2$, $a_{n+2} = a_n + (a_{n+1})^2$ for $n \geq 0$. The rest in the euclidian division of a_{2009} by 7 is:

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 5 (**E**) 6