KSF 2018 - Problems Junior (Class 9 & 10)

Time Allowed: **180 minutes**

SECTION ONE - (3 points)

1. In my family each child has at least two brothers and at least one sister. What is the smallest possible number of children in my family?

(A) 3 **(B)** 4 **(C)** 5
(D) 6 **(E)** 7

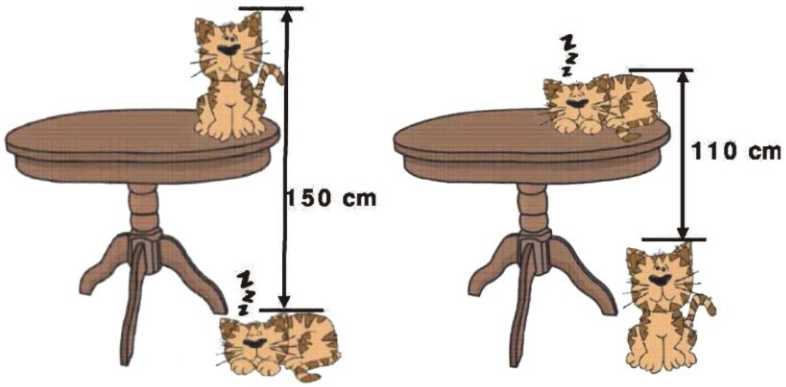
2. How many of the numbers a) $2^{100} + 2^{102}$, b) $3^{100} + 3^{102}$, c) $5^{100} + 5^{101}$, d) $9^{100} + 9^{101}$, are multiples of 10?

(A) 0 **(B)** 1 **(C)** 2
(D) 3 **(E)** 4

3. The lengths of the two sides of the triangle are 5 and 2, and the length of the third side is an odd integer number. Find the length of the third side.

(A) 3 **(B)** 4 **(C)** 5
(D) 6 **(E)** 7

4. The distance from the top of the sleeping cat on the floor to the top of the cat sitting on the table is 150 *cm*. The distance from the top of the cat sitting on the floor to the top of the cat sleeping on the table is 110 *cm*. What is the height of the table?

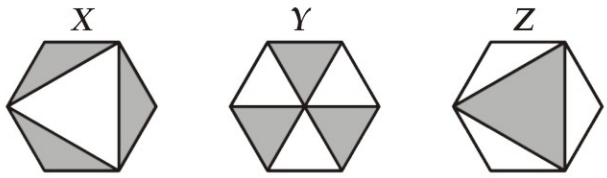


(A) 110 *cm* **(B)** 120 *cm* **(C)** 130 *cm*
(D) 140 *cm* **(E)** 150 *cm*

5. The sum of 5 consecutive integers is 10^{2018}. What is the middle number?

(A) 10^{2013} **(B)** 5^{2017} **(C)** 10^{2017}
(D) 2^{2018} **(E)** $2 \cdot 10^{2017}$

6. Given three congruent regular hexagons, we call X, Y, Z the total area of the shaded zones in each one of the figures. Which of the following statements is true?



(A) $X = Y = Z$ **(B)** $Y = Z \neq X$
(C) $Z = X \neq Y$ **(D)** $X = Y \neq Z$
(E) Each of the three areas X, Y, Z has a different value.

7. Mary has collected 42 apples, 60 apricots and 90 cherries. She wants to divide them into identical piles using all of the fruit and then give a pile to each of her friends. What is the largest number of piles can she make?

(A) 3 **(B)** 6 **(C)** 10
(D) 14 **(E)** 42

8. Some of the digits in the following correct addition have been replaced by the letters P, Q, R and S, as shown. How much is $P+Q+R+S$?

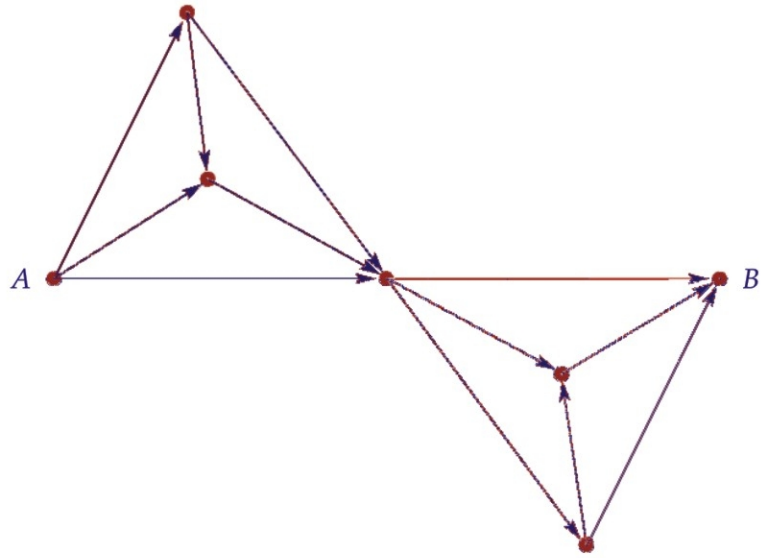
$$\begin{array}{r} P\,4\,5 \\ +\,Q\,R\,S \\ \hline 6\,5\,4 \end{array}$$

(A) 14 **(B)** 15 **(C)** 16
(D) 17 **(E)** 24

9. What is the sum of 25% of 2018 and 2018% of 25?

(A) 1009 **(B)** 2016 **(C)** 2018
(D) 3027 **(E)** 5045

10. In the picture shown you should go from A to B following the arrows. How many different routes are possible?



(A) 20 **(B)** 16 **(C)** 12
(D) 9 **(E)** 6

SECTION TWO - (4 points)

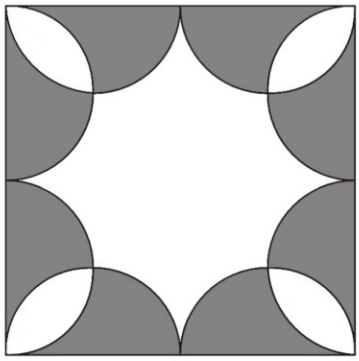
11. Two buildings are located on one street at a distance of 250 metres from each other. There are 100 students living in the first building, and there are 150 students living in the second building. Where should a bus stop be built so that the total distance that all residents of both buildings have to walk from this bus stop to their buildings would be the least possible?

(A) in front of the first building
(B) 100 metres from the first building
(C) 100 metres from the second building
(D) in front of the second building
(E) anywhere between the buildings

12. There are 105 numbers written in a row: 1,2,2,3,3,3,4,4,4,4,5,5,5,5,5,… (Each number n is written exactly n times). How many of these numbers are divisible by 3?

(A) 4 **(B)** 12 **(C)** 21
(D) 30 **(E)** 45

13. Eight congruent semicircles are drawn inside a square of length 4. What is the area of the non-shaded part of the square?



(A) 2π **(B)** 8 **(C)** $6+\pi$
(D) $3\pi-2$ **(E)** 3π

14. On a certain day 40 trains each travelled between two of the towns M,N,O,P and Q. 10 trains travelled either from or to M. 10 trains travelled either from or to N. 10 trains travelled either from or to O. 10 trains travelled either from or to P. How many trains travelled from or to Q?

(A) 0 **(B)** 10 **(C)** 20
(D) 30 **(E)** 40

15. At the University of Humanities you can study languages, history and philosophy. 35 % of students that study languages, study English. 13 % of the university students study a language other than English. No student studies more than one language. What percentage of the university students study languages?

(A) 13 % **(B)** 20 % **(C)** 22 %
(D) 48 % **(E)** 65 %

16. Peter wanted to buy a book, but he didn't have any money. He bought it with the help of his father and his two brothers. His father gave him half of the amount given by his brothers. His elder brother gave him one third of what the others gave. The younger brother gave him 10 EUR. What was the price of the book?

(A) 24 EUR **(B)** 26 EUR **(C)** 28 EUR
(D) 30 EUR **(E)** 32 EUR

17. How many 3-digit numbers are there with the property that the 2-digit number obtained by deleting the middle digit is equal to one ninth of the original 3-digit number?

(A) 1 **(B)** 2 **(C)** 3
(D) 4 **(E)** 5

18. In the calculation shown, how many times does the term 2018^2 appear inside the square root to make the calculation correct?

$$\sqrt{2018^2 + 2018^2 + \ldots + 2018^2} = 2018^{10}$$

(A) 5 **(B)** 8 **(C)** 18
(D) 2018^8 **(E)** 2018^{18}

19. How many digits does the number resulting from this calculation have?:

$$\frac{1}{9} \times 10^{2018} \times (10^{2018} - 1)?$$

(A) 2017 **(B)** 2018 **(C)** 4035
(D) 4036 **(E)** 4037

20. There are two diagonals drawn in a regular 2018-gon with its vertices numbered from 1 to 2018. One diagonal connects the vertices with the numbers 18 and 1018, the other connects the vertices with the numbers 1018 and 2000. How many vertices do the resulting three polygons have?

(A) 38, 983, 1001 **(B)** 37, 983, 1001 **(C)** 38, 982, 1001
(D) 37, 982, 1000 **(E)** 37, 983, 1002

SECTION THREE - (5 points)

21. Several integers are written on a blackboard, including the number 2018. The sum of all these integers is 2018. The product of these integers is also 2018. Which of the following could be the number of integers written on the blackboard?

(A) 2016 **(B)** 2017 **(C)** 2018
(D) 2019 **(E)** 2020

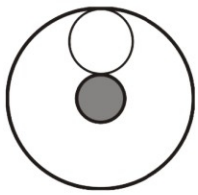
22. Four positive numbers are given. You choose three of them, calculate their arithmetic mean and then add the fourth number. This can be done in four different ways. The results are 17, 21, 23 and 29 respectively. What is the largest of the given four numbers?

(A) 12 **(B)** 15 **(C)** 21
(D) 24 **(E)** 29

23. The points A_0, A_1, A_2,... lie on a line such that $A_0A_1 = 1$ and the point A_n is the midpoint of the segment $A_{n+1}A_{n+2}$ for every non-negative integer n. What is the length of the segment A_0A_{11}?

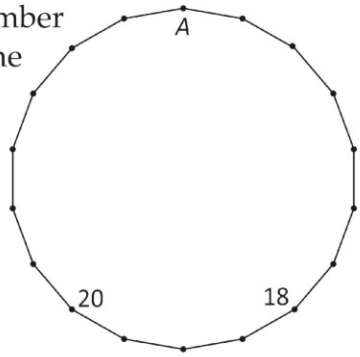
(A) 171 **(B)** 341 **(C)** 512
(D) 587 **(E)** 683

24. Two concentric circles of radii 1 and 9 make a ring. In the interior of this ring n circles are drawn without overlapping, each being tangent to both of the circles of the ring (an example of such shape for $n = 1$ and different radii is shown in the picture). What is the largest possible value for n?



(A) 1 **(B)** 2 **(C)** 3
(D) 4 **(E)** 5

25. At each vertex of the 18-gon in the picture a number should be written which is equal to the sum of the numbers at the two adjacent vertices. Two of the numbers are given. What number should be written at the vertex A?



(A) 2018 **(B)** -20
(C) 18 **(D)** 38
(E) -38

26. Diana draws a rectangular grid of 12 squares on squared paper. Some of the squares are painted black. In each blank square she writes the number of black squares that share a side with it. The figure shows an example. Now she does the same in a rectangular grid with 2018 squares. What is the maximum value that she can obtain as the result of the sum of all the numbers in the grid?

1	■	2	1
0	3	■	■
1	■	2	1

(A) 1262 **(B)** 2016 **(C)** 2018
(D) 3025 **(E)** 3027

27. Seven small cubes have been deleted from a 3 x 3 x 3 cube (see the picture). We cut this cube by the plane passing through the centre of the cube and perpendicular to one of its four big diagonals. What will the cross-section look like?

(A) 

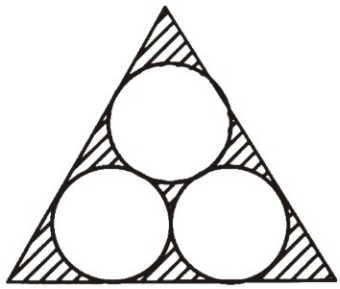
(B) 

(C) 

(D) 

(E) 

28. Given three circles each of radius 1 *cm* tangent to each other and inscribed in an equilateral triangle. What is the area of shaded region?



(A) $4\sqrt{3}-\frac{\pi}{4}$ **(B)** $2\sqrt{3}-\pi$ **(C)** $\pi-\sqrt{3}$

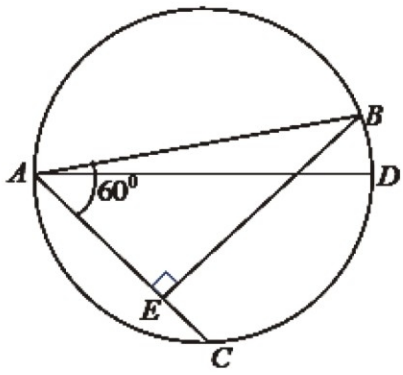
(D) $4\sqrt{3}+6-3\pi$ **(E)** $2-\sqrt{3}+\pi$

29. Ed made a large cube by gluing together a number of small identical cubes and then he painted some of the faces of the large cube. His sister Nicole dropped the cube and it broke into the original small cubes. 45 of these small cubes didn't have any painted faces. How many faces of the large cube did Ed paint?

(A) 2 **(B)** 3 **(C)** 4

(D) 5 **(E)** 6

30. Two chords AB and AC are drawn in a circle with diameter AD. The angle $\angle BAC = 60^0$, $BE \perp AC$, $EC = 3\ cm$. What is the length of the chord BD?



(A) $\sqrt{3}$ **(B)** 2 **(C)** 3
(D) $2\sqrt{3}$ **(E)** $3\sqrt{2}$