

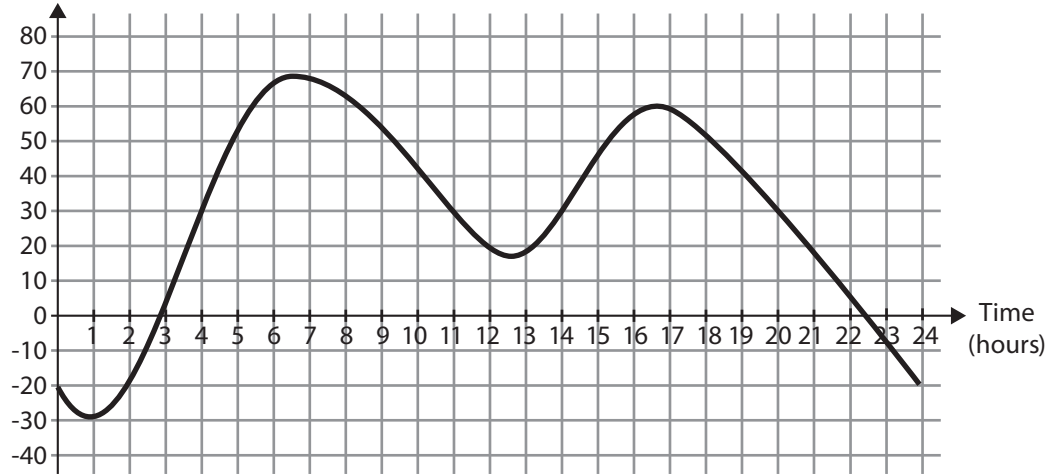
Level Student (Class 11 & 12)

Time Allowed : 3 hours

SECTION ONE - (3 points problems)

1. The water level in a port city rises and falls on a certain day as shown in the figure. For how many hours was the water level above 30 cm on that day?

Water level (cm)

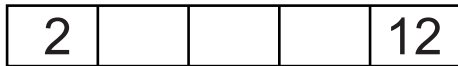


- (A) 5 (B) 6 (C) 7 (D) 9 (E) 13

2. The number $\sqrt[3]{2\sqrt{2}}$ is equal to

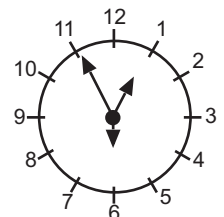
- (A) 1 (B) $\sqrt{2}$ (C) $\sqrt[6]{4}$ (D) $\sqrt[3]{4}$ (E) 2

3. In a list of five numbers, the first number is 2 and the last number is 12. The product of the first three numbers is 30, the product of the three in the middle is 90 and the product of the last three numbers is 360. Which number is in the centre of the list?



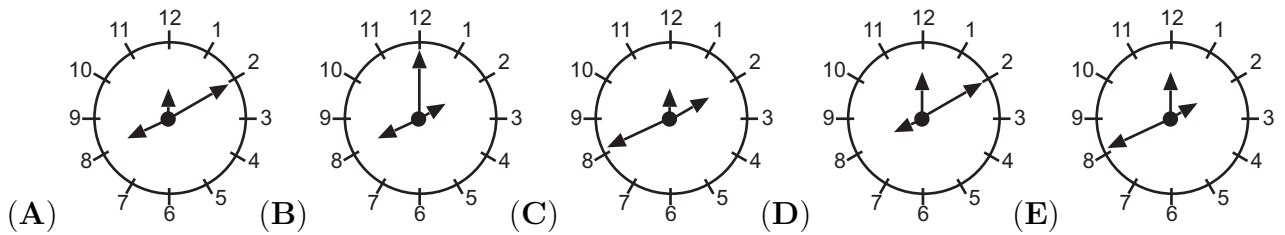
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 10

4. A clock has 3 hands of different length (for hours, for minutes, and for seconds). We do not know which hand is which, but we know that the clock runs correctly. At 12:55:30 the hands

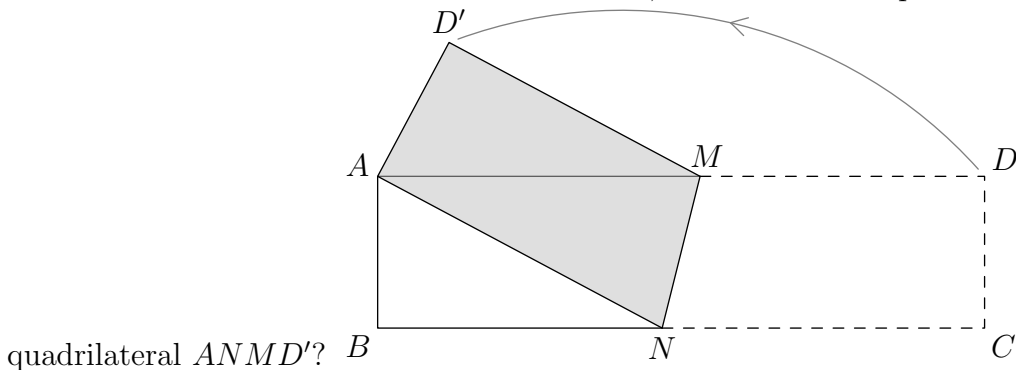


were in the positions shown. Which of the pictures shows this clock at 8:10:00?

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5. A rectangular piece of paper $ABCD$ measuring $4 \text{ cm} \times 16 \text{ cm}$ is folded along the line MN so that vertex C coincides with vertex A , as shown in the picture. What is the area of



- (A) 28 cm^2 (B) 30 cm^2 (C) 32 cm^2 (D) 48 cm^2 (E) 56 cm^2

6. The sum of the digits of a nine-digit integer is 8. What is the product of these digits?

- (A) 0 (B) 1 (C) 8 (D) 9 (E) 9!

7. The maximum value of the integer n , for which $n^{200} < 5^{300}$, is equal to:

- (A) 5 (B) 6 (C) 8 (D) 11 (E) 12

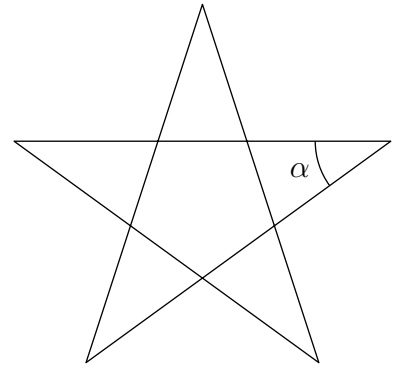
8. Which of the following functions satisfies the equation

$$f\left(\frac{1}{x}\right) = \frac{1}{f(x)}?$$

- (A) $f(x) = \frac{2}{x}$ (B) $f(x) = \frac{1}{x+1}$ (C) $f(x) = 1 + \frac{1}{x}$ (D) $f(x) = \frac{1}{x}$ (E) $f(x) = x + \frac{1}{x}$

9. A real number x satisfies $x^3 < 64 < x^2$. Which statement is correct?

- (A) $0 < x < 64$ (B) $-8 < x < 4$ (C) $x > 8$ (D) $-4 < x < 8$ (E) $x < -8$



10. What is the size of the angle α in the regular 5-pointed star?

- (A) 24° (B) 30° (C) 36° (D) 45° (E) 72°

SECTION TWO - (4 points problems)

11. My age is a two-digit integer, which is a power of 5, and my cousin's age is a two-digit integer, which is a power of 2. The sum of the digits of our ages is an odd number. What is the product of the digits of our ages?

- (A) 240 (B) 2010 (C) 60 (D) 50 (E) 300

12. A travel agency organized four optional tours of Sicily for a group of tourists. Each tour had a participation rate of 80 %. What is the smallest possible percentage of tourists who took part in all four tours?

- (A) 80% (B) 60% (C) 40% (D) 20% (E) 16%

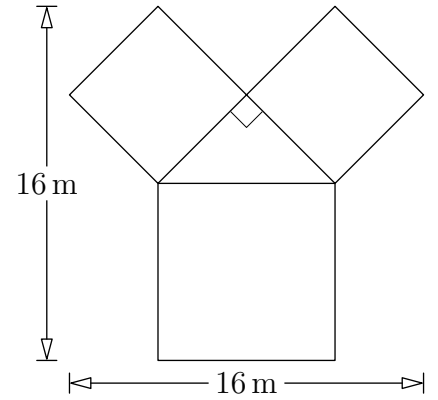
13. The set of solutions for the inequality $|x| + |x - 3| > 3$ is:

- (A) $(-\infty, 0) \cup (3, +\infty)$ (B) $(-3, 3)$ (C) $(-\infty, -3)$
(D) $(-3, +\infty)$ (E) all real numbers

14. School marks in Slovakia are divided into five degrees, from 1 (the best) to 5. In one Slovak school, a test didn't turn out very well in the 4th class. The average mark was 4. Boys did a little better, their average mark was 3.6 while the average mark of the girls was 4.2. Which of the following statements about the class is correct?

- (A) There are twice as many boys as girls. (B) There are 4 times as many boys as girls.
(C) There are twice as many girls as boys. (D) There are 4 times as many girls as boys.
(E) There are as many boys as girls.

15. The picture shows the plan of a rose bed. White roses grow in the two equal squares, and red roses grow in the third square. Yellow roses grow in the right-angled triangle. Both the



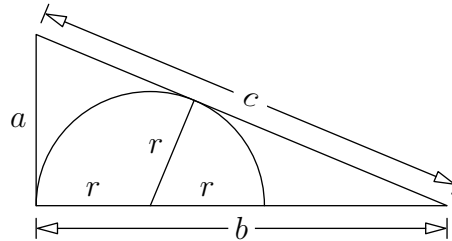
length and width of the bed are 16 m. What is the area of the rose bed?

- (A) 114 m^2 (B) 130 m^2 (C) 144 m^2 (D) 160 m^2 (E) 186 m^2

16. All the tickets for the first row in a cinema were sold. The seats are numbered consecutively starting with 1. An extra ticket was sold for one seat by mistake. The sum of the seat numbers on all tickets sold for that row is equal to 857. What is the number of the seat for which two tickets were sold?

- (A) 4 (B) 16 (C) 25 (D) 37 (E) 42

17. We are given a right-angled triangle with sides of length a , b and c . What is the radius r



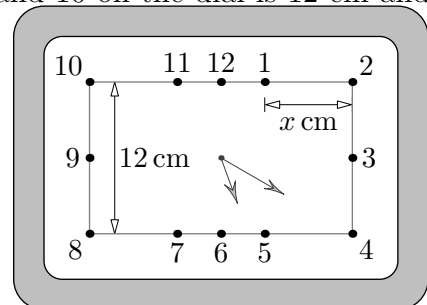
of the inscribed semicircle shown in the figure?

- (A) $\frac{a(c-a)}{2b}$ (B) $\frac{ab}{a+b+c}$ (C) $\frac{ab}{b+c}$ (D) $\frac{2ab}{a+b+c}$ (E) $\frac{ab}{a+c}$

18. A square $ABCD$ has sides of length 2. E and F are the midpoints of the sides AB and AD respectively. G is a point on CF such that $3CG = 2GF$. The area of triangle BEG is:

- (A) $\frac{7}{10}$ (B) $\frac{4}{5}$ (C) $\frac{8}{5}$ (D) $\frac{3}{5}$ (E) $\frac{6}{5}$

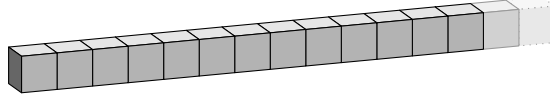
19. The clock in the picture is rectangular in shape, but each hand moves at a constant rate, like a normal clock. The distance between the numbers 8 and 10 on the dial is 12 cm and the



distance between 1 and 2 is x cm. What is the value of x ?

- (A) $3\sqrt{3}$ (B) $2\sqrt{3}$ (C) $4\sqrt{3}$ (D) $2 + \sqrt{3}$ (E) $12 - 3\sqrt{3}$

20. A kangaroo wants to build a row of standard dice (for a standard die, each pair of opposite faces has a total of 7 dots). He can glue two faces together if they have the same number of dots. He would like the total number of dots on the outer faces of the dice in the row to be



2012. How many dice does he need?

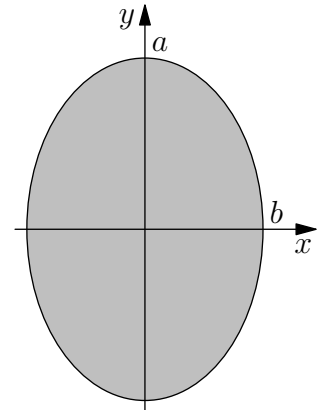
- (A) 70 (B) 71 (C) 142 (D) 143
 (E) A total of 2012 dots is impossible.

SECTION THREE - (5 points problems)

21. What is the smallest possible size of an angle in an isosceles triangle ABC that has a median that divides the triangle into two isosceles triangles?

- (A) 15° (B) 22.5° (C) 30° (D) 36° (E) 45°

22. Let $a > b$. If the ellipse shown in the picture is rotated around the x -axis one obtains the ellipsoid E_x with the volume $\text{Vol}(E_x)$. If the ellipse is rotated around the y -axis one obtains E_y



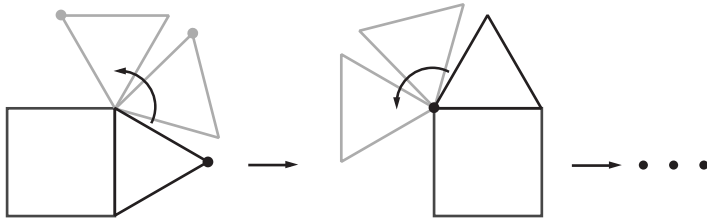
with the volume $\text{Vol}(E_y)$. Which of the following statements is true?

- (A) $E_x = E_y$ and $\text{Vol}(E_x) = \text{Vol}(E_y)$ (B) $E_x = E_y$ but $\text{Vol}(E_x) \neq \text{Vol}(E_y)$
 (C) $E_x \neq E_y$ and $\text{Vol}(E_x) > \text{Vol}(E_y)$ (D) $E_x \neq E_y$ and $\text{Vol}(E_x) < \text{Vol}(E_y)$
 (E) $E_x \neq E_y$ but $\text{Vol}(E_x) = \text{Vol}(E_y)$

23. Consider two operations which can be performed on a fraction: 1) increase the numerator by 8; 2) increase the denominator by 7. Having performed a total number of n such operations in some order, starting with the fraction $\frac{7}{8}$ we obtain a fraction of equal value. What is the smallest possible value of n ?

- (A) 56 (B) 81 (C) 109 (D) 113
 (E) This is impossible.

24. An equilateral triangle rolls without slipping around a square with side length 1 (see picture).



What is the length of the path that the marked point covers until the triangle and the point reach their starting positions the next time?

- (A) 4π (B) $\frac{28}{3}\pi$ (C) 8π (D) $\frac{14}{3}\pi$ (E) $\frac{21}{2}\pi$

25. How many permutations (x_1, x_2, x_3, x_4) of the set of integers $\{1, 2, 3, 4\}$ have the property that the sum $x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1$ is divisible by 3?

- (A) 8 (B) 12 (C) 14 (D) 16 (E) 24

26. After an algebra lesson, the following was left on the blackboard: the graph of the function $y = x^2$ and 2012 lines parallel to the line $y = x$, each of which intersects the parabola in two points. The sum of the x -coordinates of the points of intersection of the lines and the parabola is:

- (A) 0 (B) 1 (C) 1006 (D) 2012
(E) impossible to determine

27. Three vertices of a cube (not all on the same face) are $P(3, 4, 1)$, $Q(5, 2, 9)$ and $R(1, 6, 5)$. Which point is the centre of the cube?

- (A) $A(4, 3, 5)$ (B) $B(2, 5, 3)$ (C) $C(3, 4, 7)$ (D) $D(3, 4, 5)$ (E) $E(2, 3, 5)$

28. In the sequence $1, 1, 0, 1, -1, \dots$, each of the first two terms a_1 and a_2 is 1. The third term is the difference of the preceding two terms, that is, $a_3 = a_1 - a_2$. The fourth is the sum of the two preceding terms, that is, $a_4 = a_2 + a_3$. Then $a_5 = a_3 - a_4$, $a_6 = a_4 + a_5$, and so on. What is the sum of the first 100 terms of this sequence?

- (A) 0 (B) 3 (C) -21 (D) 100 (E) -1

29. Ioana picks out two numbers a and b from the set $\{1, 2, 3, \dots, 26\}$. The product ab is equal to the sum of the remaining 24 numbers. What is the value of $|a - b|$?

- (A) 10 (B) 9 (C) 7 (D) 2 (E) 6

30. Every cat in Wonderland is either wise or mad. If a wise cat happens to be in one room with 3 mad ones it turns mad. If a mad cat happens to be in one room with 3 wise ones it is exposed by them as mad. Three cats entered an empty room. Soon after the 4th cat entered,

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the 1st one went out. After the 5th cat entered, the 2nd one went out, etc. After the 2012th cat entered, it happened for the first time that one of the cats was exposed as mad. Which of these cats could both have been mad after entering the room?

- (A) The 1st one and the 2011th one (B) The 2nd one and the 2010th one
(C) The 3rd one and the 2009th one (D) The 4th one and the last one
(E) The 2nd one and the 2011th one

