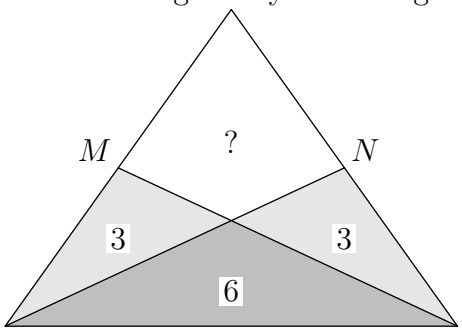
**Level Junior (Class 9 & 10)**
**Time Allowed : 3 hours**

## SECTION ONE - (3 points problems)

**1.** The diagram shows an an isoceles triangle; $M$ and $N$ are the midpoints of the equal sides. The triangle has been divided into four regions by two straight lines. Three of the regions have areas 3, 3 and 6, as shown. 

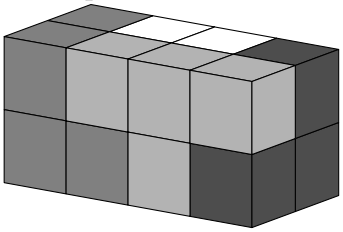
What is the area of the fourth region?

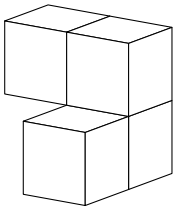
(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

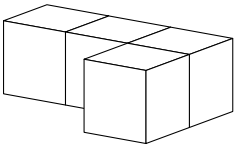
**2.** $11.11 - 1.111 =$

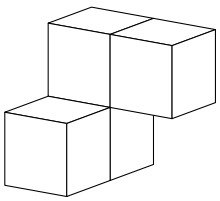
(**A**) 9.009 (**B**) 9.0909 (**C**) 9.99 (**D**) 9.999 (**E**) 10

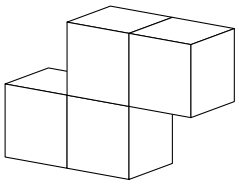
**3.** A cuboid is made of four pieces, as shown. Each piece consists of four cubes and is a single colour. What is the shape of the white piece?

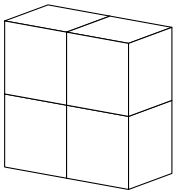


(**A**) 

(**B**) 

(**C**) 

(**D**) 

(**E**) 

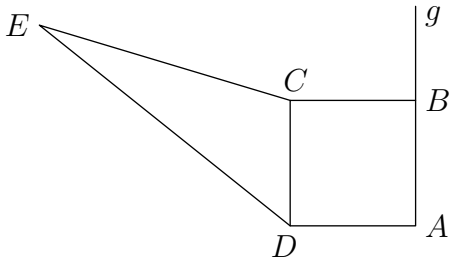
**4.** When Alice wants to send a message to Bob, she uses the following system, known to Bob. For each letter in the message, she converts the letter to a number, using $A = 01$, $B = 02$, $C = 03$, ..., $Z = 26$, and then calculates $2\times$ number $+9$. Alice sends the sequence of results to Bob. This morning Bob received the sequence $25 - 19 - 45 - 38$. What was the original

message?

(**A**) HERO (**B**) HELP (**C**) HEAR (**D**) HERS

(**E**) Alice has made a mistake.

**5.** The square $ABCE$ has side length 4 cm. The square has the same area as the triangle $ECD$. What is the distance from the point $D$ to the line $g$?




(**A**) 8 cm (**B**) $(4 + 2\sqrt{3})$ cm (**C**) 12 cm

(**D**) $10\sqrt{2}$ cm (**E**) Depends on the location of $D$

**6.** The sum of the digits of a seven-digit integer is 6. What is the product of these digits?

(**A**) 0 (**B**) 6 (**C**) 7

(**D**) $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$ (**E**) 5

**7.** $ABC$ is a right-angled triangle whose legs are 6 cm and 8 cm long. The points $K$, $L$, $M$ are the centres of the sides of the triangle. How long is the perimeter of the triangle $KLM$?

(**A**) 10 cm (**B**) 12 cm (**C**) 15 cm (**D**) 20 cm (**E**) 24 cm

**8.** In four of the following expressions we can replace each number 8 by another positive number (always using the same number for every replacement) and obtain the same result. Which expression does not have this property?

(**A**) $(8 + 8 - 8) : 8$ (**B**) $8 + (8 : 8) - 8$ (**C**) $8 : (8 + 8 + 8)$

(**D**) $8 - (8 : 8) + 8$ (**E**) $8 \cdot (8 : 8) : 8$

**9.** Two sides of a quadrilateral are equal to 1 and 4. One of the diagonals has length 2, and divides the quadrilateral into two isosceles triangles. How long is the perimeter of the quadrilateral?

(**A**) 8 (**B**) 9 (**C**) 10 (**D**) 11 (**E**) 12

**10.** Each of the numbers 144 and 220 is divided by the positive integer $N$, giving a remainder of 11 in each case. What is the value of $N$?

(**A**) 7 (**B**) 11 (**C**) 15 (**D**) 19 (**E**) 38

## SECTION TWO - (4 points problems)

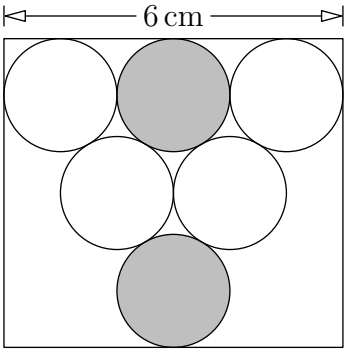
**11.** If Adam stands on the table and Mike stands on the floor, then Adam is 80 cm taller than Mike. If Mike stands on the same table and Adam is on the floor, then Mike is one metre taller than Adam. How high is the table?

(**A**) 20 cm (**B**) 80 cm (**C**) 90cm (**D**) 100 cm (**E**) 120 cm

**12.** Denis and Mary were tossing a coin. If the coin showed heads the winner was Mary and Denis had to give her 2 candies. If the coin showed tails the winner was Denis and Mary had to give him three candies. After 30 games each of them had as many candies as at the start of the game. How many times did Denis win?

(**A**) 6 (**B**) 12 (**C**) 18 (**D**) 24 (**E**) 30

**13.** A rectangle of length 6 cm encloses an "equilateral triangle" of touching circles, as shown.



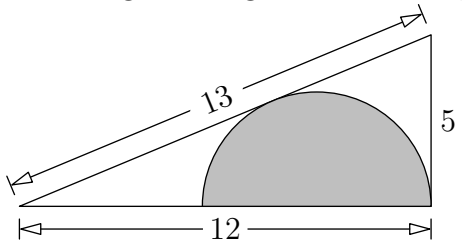

What is the shortest distance between the two grey circles?

(**A**) 1 (**B**) $\sqrt{2}$ (**C**) $2\sqrt{3}-2$ (**D**) $\frac{\pi}{2}$ (**E**) 2

**14.** In Billy's room there are four clocks. Each clock is either slow or fast. The first clock is wrong by 2 minutes, the second clock by 3 minutes, the third by 4 minutes and the fourth by 5 minutes. One day Billy wanted to know the exact time by his clocks, which read 6 minutes to 3, 3 minutes to 3, 2 minutes past 3, and 3 minutes past 3. What was the exact time then?

(**A**) 3:00 (**B**) 2:57 (**C**) 2:58 (**D**) 2:59 (**E**) 3:01

**15.** The diagram shows a right triangle with sides 5, 12 and 13. What is the radius of the inscribed semicircle?




(**A**) 7/3 (**B**) 10/3 (**C**) 12/3 (**D**) 13/3 (**E**) 17/3

**16.** How many four-digit integers are there for which the hundreds digit is 3 and the sum of the other three digits is also 3?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

**17.** Kanga is writing twelve numbers chosen from 1 to 9 in the cells of a $4 \times 3$ grid, so that the sum of every row is the same and the sum of every column is the same. Kanga has already written some of the numbers, as shown. What number should be written in the shaded square?

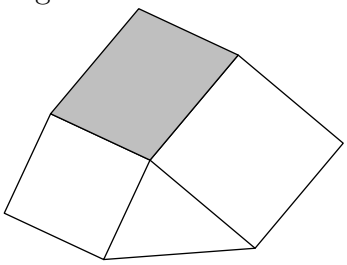
| | | | |
|---|---|---|---|
| 2 | 4 | | 2 |
| | 3 | 3 | |
| 6 | | 1 | |

(**A**) 1 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 9

**18.** Three athletes Kan, Ga and Roo took part in a Marathon race. Before the race, four spectators discussed the athletes' chances. The first said: "Either Kan or Ga will win". The second said: "If Ga is the second, then Roo will win". The third said: "If Ga is the third, then Kan will not win". The fourth said: "Either Ga or Roo will be the second". After the race it turned out that all four statements were true. Kan, Ga and Roo were the three top athletes in the race. In what order did they finish?

(**A**) Kan, Ga, Roo (**B**) Kan, Roo, Ga (**C**) Roo, Ga, Kan
(**D**) Ga, Roo, Kan (**E**) Ga, Kan, Roo

**19.** The diagram shows a shape formed from two squares with sides 4 and 5 cm, a triangle with area 8 cm$^2$ and a shaded parallelogram. What is the area of the parallelogram?



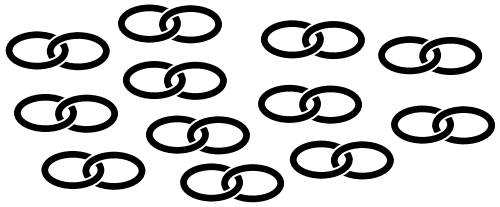
(**A**) 15 cm$^2$ (**B**) 16 cm$^2$ (**C**) 18 cm$^2$ (**D**) 20 cm$^2$ (**E**) 21 cm$^2$

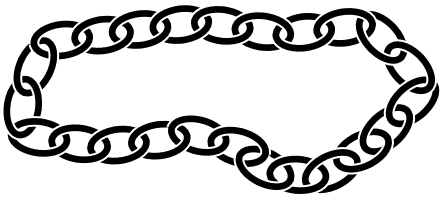
**20.** Ann has written $2012 = m^m \cdot (m^k - k)$ for some positive integer values of $m$ and $k$. What is the value of $k$?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 9
(**E**) 11

## SECTION THREE - (5 points problems)

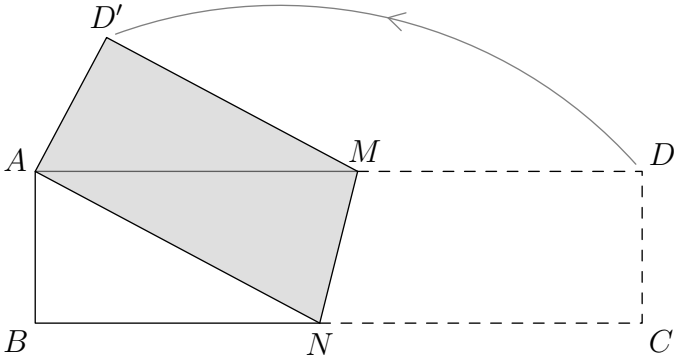
**21.** A jeweller has 12 pieces of chain, each with two links. He wants to make one big closed necklace of them, as shown. To do this he has to open some links (and close them afterwards). What is the smallest number of links he has to open?





(**A**) 8 (**B**) 9 (**C**) 10 (**D**) 11 (**E**) 12

**22.** A rectangular piece of paper $ABCD$ measuring 4 cm × 16 cm is folded along the line $MN$ so that vertex $C$ coincides with vertex $A$, as shown. What is the area of the pentagon $ABNMD'$?




(**A**) 17 (**B**) 27 (**C**) 37 (**D**) 47 (**E**) 57

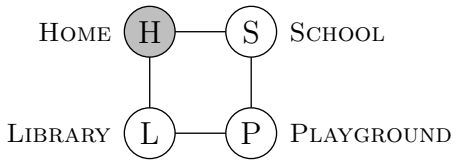
**23.** Train G passes a milestone in 8 seconds before meeting train H. The two trains pass each other in 9 seconds. Then train H passes the milestone in 12 seconds. Which of the following statements about the lengths of the trains is true?

(**A**) G is twice as long as H (**B**) G and H are of equal length (**C**) H is 50 % longer than G
(**D**) H is twice as long as G (**E**) Nothing can be deduced about the lengths

**24.** The last non-zero digit of the number $K = 2^{59} \cdot 3^4 \cdot 5^{53}$ is

(**A**) 1 (**B**) 2 (**C**) 4 (**D**) 6 (**E**) 9

**25.** Peter creates a Kangaroo game. The diagram shows the board for the game. At the start, the Kangaroo is at the School $S$. According to the rules of the game, from any position except Home $H$ the Kangaroo can jump to either of the two neighboring positions. When the Kangaroo lands on $H$ the game is over. In how many ways can the Kangaroo move from $S$ to $H$ in exactly 13 jumps?




(**A**) 12 (**B**) 32 (**C**) 64 (**D**) 144 (**E**) 1024

**26.** You are given 5 lamps, each of which can be switched to "on" or "off". Each time you switch any lamp, you change its status; moreover, the status of exactly one other randomly chosen lamp is also changed. (For the same lamp, the choice of the other lamp may be different each time.) At the beginning, all the lamps are off. Then you make 10 such switch operations. Which of the following statements is now true?

(**A**) It is impossible for all the lamps to be off.

(**B**) All the lamps are definitely on.

(**C**) It is impossible for all the lamps to be on.

(**D**) All the lamps are definitely off.

(**E**) None of the statements A to D is correct.

**27.** Six different positive integers are given, the biggest of them being $n$. There exists exactly one pair of these integers such that the smaller number does not divide the bigger one. What is the smallest possible value of $n$?

(**A**) 18 (**B**) 20 (**C**) 24 (**D**) 36 (**E**) 45

**28.** Nick wrote down all three-digit integers and for each of them he wrote down the product of its digits. After that Nick found the sum of all these products. What total should Nick obtain?

(**A**) 45 (**B**) $45^2$ (**C**) $45^3$ (**D**) $2^{45}$ (**E**) $3^{45}$

**29.** The numbers from 1 to 120 have been written into 15 rows, in the manner indicated in the diagram. For which column (counting from the left) is the sum of the numbers the largest?

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | ⋯ | |
| 2 | 3 | | | | | | ⋯ | |
| 4 | 5 | 6 | | | | | ⋯ | |
| 7 | 8 | 9 | 10 | | | | ⋯ | |
| 11 | 12 | 13 | 14 | 15 | | | ⋯ | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | | ⋮ |
| 106 | 107 | 108 | 109 | 110 | 111 | 112 | ⋯ | 120 |

(**A**) 1 (**B**) 5 (**C**) 7 (**D**) 10 (**E**) 13

**30.** Let $A, B, C, D, E, F, G, H$ be the eight vertices of a convex octagon, taken in order. Randomly choose a vertex from $C, D, E, F, G, H$ and draw the line segment connecting it with vertex $A$. Once more, randomly choose a vertex from the same six vertices, but now draw the line segment connecting it with vertex $B$. What is the probability that the octagon is cut into exactly three regions by these two line segments?

(**A**) $\frac{1}{6}$ (**B**) $\frac{1}{4}$ (**C**) $\frac{4}{9}$ (**D**) $\frac{5}{18}$ (**E**) $\frac{1}{3}$