## Student: Class (11-12)

## 3-Point-Problems

**1.** Which of the following numbers is greatest?
A) 2006×2006    B) 2005×2007    C) 2004×2008
D) 2003×2009    E) 2002×2010

**2.** How many zeroes does the product of the first 2006 prime numbers end with?
A) 0    B) 1    C) 2    D) 9    E) 26

**3.** We consider the perimeter and the area of the region corresponding to the grey squares. How many more squares can we colour grey for the grey area to increase without increasing its perimeter?
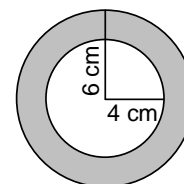A) 0    B) 7    C) 18    D) 12    E) 16
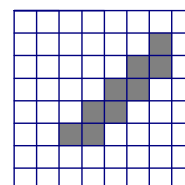
**4.** There are four cards on the table as in the figure. Every card has a letter on one side and a number on the other side. Raza said: "For every card on the table it is true that if there is a vowel on one side, there is an even number on the other side." What is the smallest number of cards Sara must turn in order to check whether Raza said the truth?
A) none    B) 1    C) 2    D) 3    E) 4

**5.** Two trains with the same length are travelling in opposite directions. The first is travelling at 100 km/h and the second at 120 km/h. A passenger on the second train observes that it takes exactly 6 sec first train to pass completely in front of him. How long does it take for a passenger on the first train to see the second train pass completely?
A) 5 sec    B) 6 sec    C) between 6 and 7 sec
D) 7 sec    E) more than 7 sec

**6.** Ahmad has two pendants made of the same material. They are equally thick and weigh the same. One of them has the shape of an annulus created from two concentric circles with the radii 6 cm and 4 cm (see the diagram). The second has the shape of a solid circle. What is the radius of the second pendant?
A) 4 cm    B) $2\sqrt{6}$ cm    C) 5 cm    D) $2\sqrt{5}$ cm    E) $\sqrt{10}$ cm

**7.** The difference between any two consecutive numbers on the list a, b, c, d, e is the same. If b=5.5 and e=10, what is the value of a?
A) 0.5    B) 3    C) 4    D) 4.5    E) 5

**8.** If $4^x=9$ and $9^y=256$, then $xy$ equals
A) 2    B) 4    C) 10    D) 36    E) 48

**9.** Consider all 9-digit integers made by using all the digits 1,2,…,9. Write each such number on a separate sheet and put all the resulting sheets in a box. What is the minimum number of sheets that you must extract from the box if you want to be certain that there are at least two numbers with the same digit in the first place among the chosen numbers?
A) 9!            B) 8!            C) 72            D) 10            E) 9

**10.** In the diagram, $AB$ has length 1; $\angle ABC = \angle ACD = 90°$; $\angle CAB = \angle DAC = \theta$. What is the length of $AD$?

A) $\cos\theta + \tan\theta$       B) $\dfrac{1}{\cos 2\theta}$       C) $\cos^2\theta$

D) $\cos 2\theta$       E) $\dfrac{1}{\cos^2\theta}$
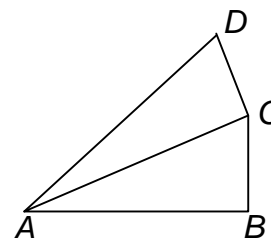
## 4-Point-Problems

**11.** Which of the following gives the formula of a function whose graph has the $y$-axis as an axis of symmetry?
A) $y = x^2 + x$       B) $y = x^2\sin x$       C) $y = x\cos x$       D) $y = x\sin x$       E) $y = x^3$

**12.** On a fair roulette wheel there are 37 numbers: 0 and the positive integers from 1 to 36. What is the probability that the ball lands on a prime number?
A) 5/18            B) 11/37            C) 11/36            D) 12/37            E) 1/3

**13.** The remainder of the division of the number 1001 by some one-digit number is equal to 5. What is the remainder of the division of the number 2006 by the same one-digit number?
A) 2            B) 3            C) 4            D) 5            E) 6

**14.** The radius of the traffic sign is 20 cm. Each of the dark pieces is a quarter of a circle. The area of all 4 quarters equals that of the light part of the sign. What is the radius of this circle?
A) $10\sqrt{2}$ cm       B) $4\sqrt{5}$ cm       C) 20/3 cm       D) 12,5 cm       E) 10 cm
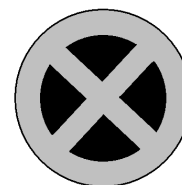
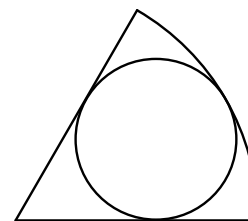**15.** You are given three prime numbers $a, b, c$ with $a > b > c$. If $a + b + c = 78$ and $a - b - c = 40$ then $abc =$
A) 438            B) 590            C) 1062            D) 1239            E) 2006

**16.** The ratio of the radii of the sector and the incircle in the picture is 3:1. Than the ratio of their areas is:
A) 3:2            B) 4:3            C) 5:3            D) 6:5            E) 5:4

**17.** Sixteen teams play in a volleyball league. Each team plays one game against each other team. For each game the winning team gets 1 point and the losing team 0 points. There are no draws. After all games have been played the team scores form an arithmetic sequence. How many points has the team in last place received?
A) 3        B) 2        C) 1        D) 0        E) none of this

**18.** Last year there were 30 more boys than girls in the school singing team. This year the members of singing team have increased by 10%: the number of girls has increased by 20% and the number of boys by 5%. How many members the singing team has this year?
A) 88        B) 99        C) 110        D) 121        E) 132

**19.** The cells of a 4×4 table are coloured black and white as shown in Fig. 1. One move allows us to exchange any two cells positioned in the same row or in the same column. What is the least number of moves necessary to obtain Fig. 2?

A) this is not possible        B) 2        C) 3        D) 4        E) 5
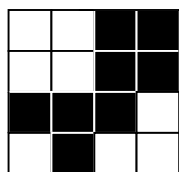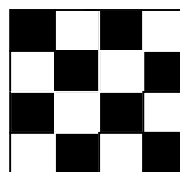


Fig. 1        Fig. 2

**20.** In a certain home there is a rose window as in the figure, where the letters R, G and B represent glass of red colour, green colour and blue colour, respectively. Knowing that 400 cm$^2$ of green glass have been used, how many cm$^2$ of blue glass are necessary?
A) 396        B) 400        C) $120\pi$        D) $90\sqrt{2}\pi$        E) 382
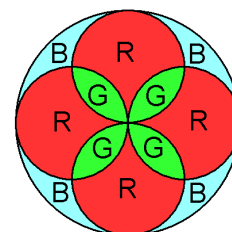


## 5-Point-Problems

**21.** Given that $a$ and $b$ are both numbers greater than 1, which of the following fractions has the greatest value?
A) $\dfrac{a}{b-1}$        B) $\dfrac{a}{b+1}$        C) $\dfrac{2a}{2b+1}$        D) $\dfrac{2a}{2b-1}$        E) $\dfrac{3a}{3b+1}$

**22.** The lengths of the sides of triangle *XYZ* are 8 cm, 9 cm and $\sqrt{55}$ cm. Find the length of the diagonal *XA* of the rectangular parallelepiped in the figure.
A) $\sqrt{90}$ cm        B) 10 cm        C) $\sqrt{120}$ cm
D) 11 cm        E) $\sqrt{200}$ cm

**23.** For how many values of the real number $b$ does the equation $x^2 - bx + 80 = 0$ have two different positive even integer solutions?
A) 0       B) 1       C) 2       D) 3       E) infinitely many

**24.** How many nonempty subsets of {1, 2, 3… 12} exist in which the sum of the largest element and the smallest element is 13?
A) 1024       B) 1175       C) 1365       D) 1785       E) 4095

**25.** Points $M$ and $N$ are given on the sides $AB$ and $BC$ of a rectangle $ABCD$. Then the rectangle is divided into several parts as shown in the figure. The areas of 3 parts are also given in the figure. Find the area of the quadrilateral marked with "?".
A) 20       B) 21       C) 25       D) 26
E) none of this.

**26.** A test is composed of ten questions each of which must be answered "a" or "b". If you answer any 5 questions „a" and any 5 questions „b", the number of correct answers is guaranteed to be at least 4. How many answer keys are there with this property?
A) $5^5$       B) 252       C) 2       D) 10       E) 22

**27.** Ali removed one number from ten consecutive natural numbers. The sum of the remaining ones is 2006. The removed number is
A) 218       B) 219       C) 220       D) 225       E) 227

**28.** In how many ways can all the numbers 1, 2, 3, 4, 5, 6 be written in the squares of the figure (one in each square) so that there are no adjacent squares in which the difference of the numbers written is equal to 3? (Squares that share only a corner are not considered adjacent.)
A) 3 x $2^5$       B) $3^6$       C) $6^3$       D) 2 x $3^5$       E) 3 x $5^2$

**29.** A die is in the position shown in the figure. It can be rolled along the path of 12 squares as shown. How many times must the die go around the path in order for it to return to its initial position with all faces in the initial positions?
A) 1       B) 2       C) 3
D) 4       E) none of this.

**30.** If each side of the regular hexagon has length $\sqrt{3}$ and XABC and XPQR are squares, what is the area of the shaded region?

A) $\dfrac{5-\sqrt{3}}{4}$       B) $\dfrac{\sqrt{3}+1}{2}$       C) $\dfrac{\sqrt{3}}{4}$

D) $\dfrac{2-\sqrt{3}}{4}$       E) $\dfrac{2+\sqrt{3}}{4}$